Exam Waves and Optics - 25 January 2016 – 9:00-12:00

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This exam contains 5 questions on 4 pages.

A few **preliminary remarks**:

- Please answer questions 4 & 5 on another (double) sheet of paper than questions 1, 2 & 3.
- Put your name and student number at the top of all sheets.
- Put your student card at the edge of the desk for checking by the assistants and show it when handing in your test.
- Add the units to the numbers calculated.

Question 1 (6 points): Fresnel equations, transmittance

A beam of light in air strikes the surface of a smooth piece of plastic (with index of refraction 1.5) at an angle of incidence of 30 degrees. The incident light has an electric field component parallel to the plane of incidence with an amplitude of 10.0 V/m and a component perpendicular to the plane of incidence with an amplitude of 20 V/m.

Questions:

a. Calculate the **amplitude** of the **parallel** component of the electric field of the **reflected** beam.

b. Calculate the **perpendicular** component of the **transmittance**.

Make use of the appropriate Fresnel equations given below.

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad (4.34)$$

$$t_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad (4.35)$$

$$r_{\|} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \qquad (4.40)$$

$$t_{\|} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \qquad (4.41)$$

For question b), in case you need to derive the appropriate equation of the transmittance, the following expression for the irradiance I of a harmonic electromagnetic wave with amplitude E_0 may be useful:

$$I = \frac{c\,\varepsilon_0}{2} E_0^2$$

Question 2 (5 points): diffraction at a single slit

The Fraunhofer diffraction pattern (irradiance) of a single slit is given by:

$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$ with $\beta = \frac{kb}{2}\sin\theta$ and k the propagation number of the light wave and b the slit width

A narrow single slit (in vacuum) in an opaque (non-transparent) screen is illuminated by plane wave light with a wavelength of 632 nm. It is observed that the center of the tenth minimum in the Fraunhofer diffraction pattern lies at an angle θ of 5.4 degrees off the central axis.

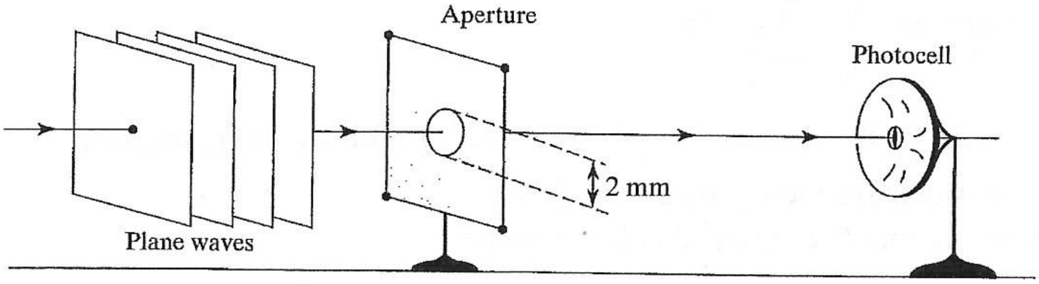
Questions:

a. What is the slit width ?

b. At what angle will the tenth minimum appear if the entire arrangement is immersed in water (index of refraction = 1.33) ?

Question 3 (5 points): Fresnel zones

A 2 mm diameter circular aperture in an opaque (non-transparant) screen is illuminated by plane waves with wavelength $\lambda = 485$ nm. A small photocell is moved along the central axis, recording the irradiance of the diffracted light. The figure below illustrates the situation.



In this situation, the radius of the m-th Fresnel zone can be approximated as: $R_m = \sqrt{m\, r_0\, \lambda}$, with r_0 the distance of the photocell to the aperture.

Questions:

a. At what minimum distance from the aperture does the photocell need to be such that Fraunhofer diffracton occurs ?

b. As the photocell approaches the aperture along the central axis, starting at a distance of 3 m from the aperture, maxima and minima will be observed. What is the distance from the aperture of the second maximum and of the third minimum ?

Note: use a new (double) sheet of paper for questions 4 & 5.

Question 4 (7 points): the mechanical oscillator model

The mechanical oscillator model is used to understand dispersion, the phenomenon whereby the index of refraction of a material depends on the frequency of an electromagnetic wave.

Questions:

a. What is the mechanical oscillator model ? Explain briefly in words.

b. Derive a dispersion equation using the following information:

1. When an external electric field $\vec{E}$ is applied to a dielectric material, the resultant dipole moment per unit volume, the so-called electric polarization $\vec{P}$, is given by:

$$\vec{P} = \left(\varepsilon - \varepsilon_0\right)\vec{E},$$

with ε_0 the permittivity of the vacuum and ε the permittivity of the dielectric material.

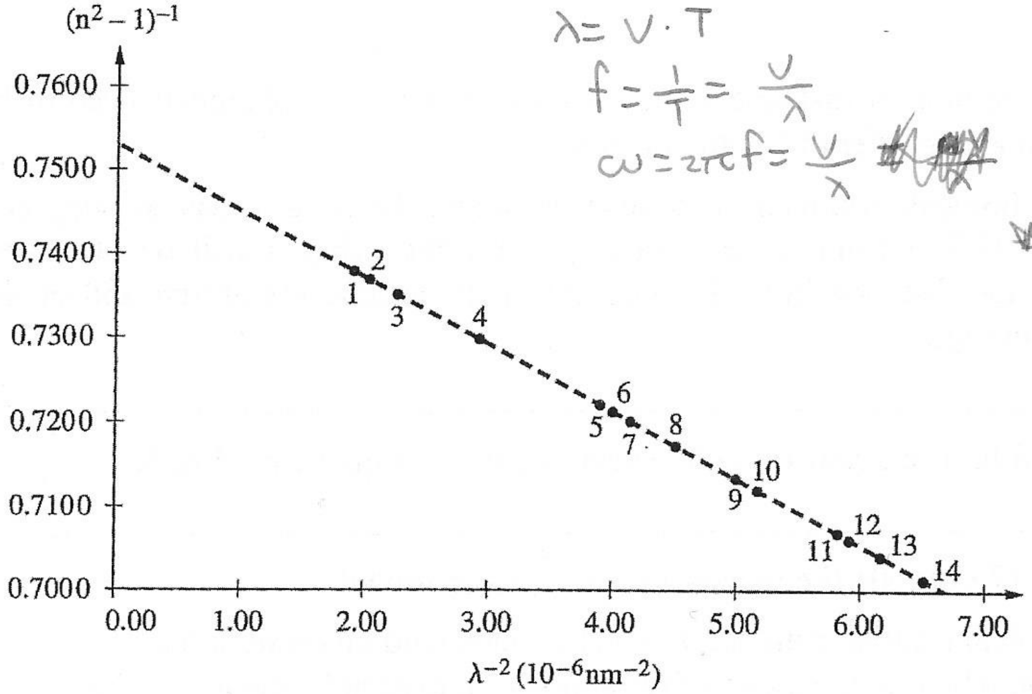
2. If a harmonic electric field $E(t)$ makes a charge q_e with mass m_e to oscillate, the displacement $x(t)$ of the charge is given by:

$$x(t) = \frac{q_e / m_e}{\left(\omega_0^2 - \omega^2\right)} E(t),$$

with ω the angular frequency of $E(t)$ and ω_0 the resonance angular frequency of the oscillator.

3. Assume that the material is non-magnetic.

Hint: the dispersion relation explains the following figure that was discussed during the lectures. (n is the index of refraction and λ is the wavelength of the light)



Question 5 (7 points): Circularly polarized light

Show how right-circularly polarised light can be obtained from the superposition of two linearly polarised harmonic waves with perpendicular polarisation directions. First give the expressions for the two individual waves and their superposition. Then show that this superposition represents right-circularly polarised light.